Conclusions

The important conclusions are as follows. The adaptive control function technique can be applied to design adaptive multipoint controllers that can be proven stable under certain ideal conditions. These systems are remarkably simple to mechanize and can incorporate such practical modifications as filters for signal conditioning and/or noise suppression. Application of this technique to a practical lateral flight control problem demonstrated its effectiveness, performance, and its practical advantages.

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Quasi-Optimum Design of an Aircraft Landing Control System

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An explicit quasi-optimum control law for longitudinal aircraft motion with particular application to the landing phase is obtained by use of Friedland's[1,2] quasi-optimum control technique. A worst-case simulation study (in which the initial errors in altitude, pitch, and angle of attack are −100 ft, −5.4°, and −2.2°, respectively) was performed for a typical jet transport. The resulting trajectory indicated that the aircraft is returned to the desired nominal trajectory in about 20 sec and does not deviate from the nominal trajectory by more than −130 ft. Simulated performance in the presence of steady wind disturbances was also determined to be satisfactory.

1. Introduction

DESPITE the rapid development in the modern optimal feedback control theory and its various applications in the control of space-oriented systems, its potential application to the control of aircraft appears relatively unexplored. The main reason for the lack of enthusiasm may be attributed in part to the fact that any realistic formulations of aircraft control problems unavoidably result in complicated mathematical expressions which are difficult to handle.

In the present paper, attention is focused on the control of aircraft elevator during the final phases of landing. The problem of aircraft landing control with quadratic performance criteria was previously reported by Ellert and Merriam,[3] who propose to guide a landing aircraft to a certain prescribed ideal track at the end of a prespecified period of time. Various performance requirements are then satisfied by a careful adjustment of coefficients in a quadratic performance integral. In this paper, the problem is formulated and approached from a somewhat different point of view. With the designation of an ideal landing trajectory y^*, a nominal elevator deflection δ^* is defined to guide the aircraft to fly along the prespecified path when no deviation therefrom is present. The nominal control is a fixed function of the nominal trajectory and, therefore, is a fixed function of the aircraft position in space. Any deviation of the aircraft dynamic variables from the values defined by the nominal trajectories is regarded as path errors. The control δ, in the presence of path errors is necessarily different from that of the nominal control and the difference $\Delta\delta = \delta - \delta^*$ is to be designed to eliminate the path errors in a manner specified by the performance index. The design of the error correction control is accomplished by applying the quasi-optimum control technique developed by Friedland,[1,2] which results in a feedback control law. Thus, if the aircraft enters the final phase of landing with its altitude, pitch angle, pitch rate and angle of attack different from specified values, the control system is to eliminate these deviations and to maintain the deviation-free condition thereafter. Since the control law is in the form of a feedback configuration, deviations due to outside disturbance are also corrected.

One convenience the quasi-optimum control technique offers is that nonlinearities can be handled effectively to allow realistic formulation of the problem. Moreover, application of the technique yields a feedback control law in closed form that can be readily mechanized.

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2. System Dynamics for Aircraft Landing

The landing problem considered in this paper is concerned with the last few hundred feet of descent. It is assumed that the aircraft is properly guided into the final phase by some other means so that only the longitudinal motion need be considered during the final approach. It is assumed also that during the period of concern an ideal descending path $y^*(\tau)$ (see Fig. 1) is specified and the variations of the pitch angle $\theta(\tau)$, the angle of attack $\alpha(\tau)$, and the aircraft forward speed $v(\tau)$ are kept small.

With these assumptions, the longitudinal motion of an aircraft is governed entirely by the elevator deflection $\delta(\tau)$. The control problem is formulated to obtain an optimum feedback law for the elevator to guide the aircraft back to the ideal descending route if any deviation therefrom is detected.

The motion of an aircraft under the previous assumptions is governed by the following dynamic equations:

$$\dot{y}(\tau) = v \sin\varphi \tag{1}$$

$$\ddot{\theta}(\tau) = C_{m0} + C_{m\dot\theta}\dot\theta + C_{m\dot\theta\dot\theta}\dot\theta|\dot\theta| + C_{m\alpha}\alpha + C_{m\dot\alpha}\dot\alpha + C_{m\delta}\delta \tag{2}$$

$$\dot\alpha(\tau) = \dot\theta - (C_{L0} + C_{L\alpha}\alpha) + (g/v) \tag{3}$$

$$\varphi(\tau) = \theta(\tau) - \alpha(\tau) \tag{4}$$

where $y(\tau)$ = instantaneous altitude of aircraft with respect to ground, $\theta(\tau)$ = instantaneous pitch angle, $\alpha(\tau)$ = instantaneous angle-of-attack, $\varphi(\tau)$ = instantaneous glide angle, $\delta(\tau)$ = instantaneous elevator deflection, v = constant aircraft speed, g = constant gravitational acceleration, C_{m0} = steady aerodynamic moment, $C_{m\dot\theta}$ = pitch damping, $C_{m\dot\theta\dot\theta}$ = nonlinear pitch damping, $C_{m\alpha}$ = angle-of-attack static stability, $C_{m\dot\alpha}$ = angle-of-attack damping, $C_{m\delta}$ = control effectiveness, C_{L0} = steady aerodynamic lift, $C_{L\alpha}$ = vertical damping.

Obviously, in order for an aircraft to fly along certain prespecified path $y^*(\tau)$, an appropriate nominal control $\delta^*(\tau)$ must be applied. Let $\theta^*(\tau)$, $\alpha^*(\tau)$, and $\varphi^*(\tau)$ denote the pitch, the angle of attack and the glide angle, respectively, along the path y^*, resulting from the application of δ^*. Then at any instant, the equations of motion (1–4) are satisfied by y^*, θ^*, α^*, ψ^*, and δ^*, and from which, the nominal control δ^* can be solved in terms of the nominal trajectory y^* to yield:

$$\varphi^* = \sin^{-1}(v/\dot y^*),$$
$$\alpha^* = (1/C_{L\alpha})[\dot\varphi^* - C_{L0} + (g/v)] \tag{5}$$
$$\theta^* = \varphi^* + (1/C_{L\alpha})[(\dot\varphi^* - C_{L0} + (g/v)]$$

$$C_{m\delta}\delta^* = -C_{m0} + (1/C_{L\alpha})\dddot{\varphi}^* + [1 - (C_{m\dot\theta} + C_{m\dot\alpha})/C_{L\alpha}]\ddot\varphi^* - [C_{m\dot\theta} + (C_{m\alpha}/C_{L\alpha})]\dot\varphi^* - C_{m\dot\theta\dot\theta}[\dot\varphi^* + (1/C_{L\alpha})\ddot\varphi^*]|\dot\varphi^* + (1/C_{L\alpha})\ddot\varphi^*| - (C_{m\alpha}/C_{L\alpha})[-C_{L0} + (g/v)] \tag{6}$$

Let Δy, $\Delta\theta$, $\Delta\alpha$, $\Delta\varphi$, and $\Delta\delta$ represent the deviations of y, θ, α, φ, and δ from their nominal values y^*, θ^*, α^*, φ^*, and δ^*, respectively, i.e.,

$$y(\tau) = y^*(\tau) + \Delta y(\tau), \qquad \theta(\tau) = \theta^*(\tau) + \Delta\theta(\tau),$$
$$\alpha(\tau) = \alpha^*(\tau) + \Delta\alpha(\tau) \tag{7}$$
$$\varphi(\tau) = \varphi^*(\tau) + \Delta\varphi(\tau), \qquad \delta(\tau) = \delta^*(\tau) + \Delta\delta(\tau)$$

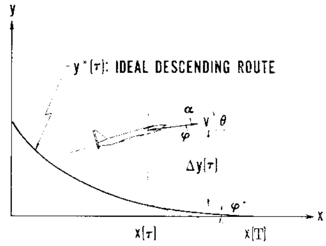
Substituting (3) into (2) to eliminate $\dot\alpha$ in (2), and then substituting (4–7) into the resulting expression and into (1), (3) results in

$$\Delta\dot y = (v\cos\varphi^*)\sin(\Delta\theta - \Delta\alpha) \tag{8}$$

$$\Delta\ddot\theta = (C_{m\dot\theta} + C_{m\dot\alpha})\Delta\dot\theta + C_{m\dot\theta\dot\theta}\Delta\dot\theta|\Delta\dot\theta| + (C_{m\alpha} - C_{m\dot\alpha}C_{L\alpha})\Delta\alpha + C_{m\delta}\Delta\delta \tag{9}$$

$$\Delta\dot\alpha = \Delta\dot\theta - C_{L\alpha}\Delta\alpha \tag{10}$$

Fig. 1 Aircraft landing scheme.



In arriving at (8), it is assumed that $\sin\varphi^*[\cos(\Delta\theta - \Delta\alpha) - 1] \approx 0$.

It is observed that (7) effectively separates the control δ into two mutually independent parts; the nominal control δ^* and the error correction control $\Delta\delta$. The nominal control δ^* is a fixed function of the ideal descending route y^* and is given by (6). The error correction control $\Delta\delta$, however, is almost independent of the ideal route, since as can be seen from (8), for any physically reasonable value of φ^*, i.e., $0 > \varphi^* > -5°$, the approximation $\cos\varphi^* \approx 1$ always holds. In what follows, our subsequent attention will be focused to the design of the error correction control $\Delta\delta$.

The primary task of designing a control system is to provide an optimum elevator control law so that any initial deviations of aircraft variables can be eliminated rapidly while maintaining the rate of descent, the variations of pitch angle, angle of attack and elevator angle, within certain permissible values. Since these flight requirements are directly proportional to the torque generated by the elevator movement, it is appropriate to choose the performance index of the form

$$S = \int_t^T \left[k + \frac{1}{2}(C_{m\delta}\Delta\delta)^2\right]d\tau \tag{11}$$

where the weighting factor k is assumed constant for simplicity. Adjustment of the weighting k enables the control system to provide the torque that produces a satisfactory flight between the two extreme responses: fast but rough and slow but smooth. The terminal time T is the time at which all deviations are reduced to zero.

In accordance with the state space description of dynamic systems, the state variables are defined by

$$x_1(t) = \Delta y(t), \qquad x_2(t) = \Delta\theta(t)$$
$$x_3(t) = \Delta\dot\theta(t), \qquad x_4(t) = \Delta\alpha(t) \tag{12}$$

If we further let

$$V = v\cos\varphi^*,\ a_1 = -(C_{m\dot\theta} + C_{m\dot\alpha}),\ a_2 = -C_{m\dot\theta\dot\theta}$$
$$b_1 = -(C_{m\alpha} - C_{m\dot\alpha}C_{L\alpha}),\ b_2 = C_{L\alpha},\ u(t) = C_{m\delta}\Delta\delta(t) \tag{13}$$

then the equations of motion (8–10) in state space form can be written as

$$\dot x_1 = V\sin(x_2 - x_4) \tag{14}$$

$$\dot x_2 = x_3 \tag{15}$$

$$\dot x_3 = -a_1x_3 - a_2x_3|x_3| - b_1x_4 + u \tag{16}$$

$$\dot x_4 = x_3 - b_2x_4 \tag{17}$$

and the performance index (11) becomes

$$S = \int_t^T \left(k + \frac{1}{2}u^2\right)d\tau \tag{18}$$

where the terminal time T is unspecified and $x_1(T) = x_2(T) = x_3(T) = x_4(T) = 0$.

3. Design of Quasi-Optimum Feedback Control Law

The basis of the quasi-optimum control technique to be employed in solving the problem is the observation that a complex process can often be approximated by a much simpler

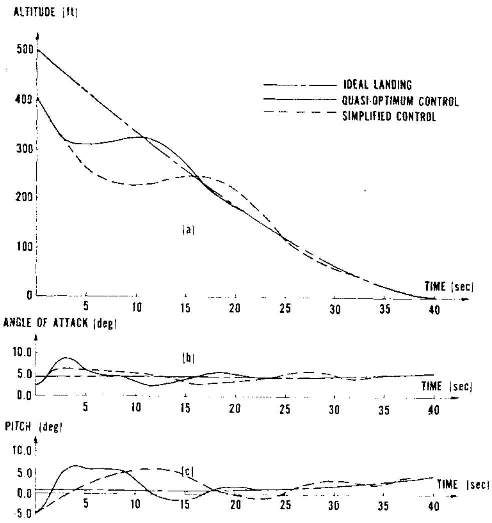


Fig. 2 Transient responses for $k = 0.001$.

model for which the exact optimum feedback control law can be expressed in closed form. This simplified control law is then corrected to account for the difference between the original process and its simplified model. This approach is practical, because the required correction can be computed without prior knowledge of the exact control law for the original process.

Following the development in Ref. 1, the problem is first formulated in the manner of Pontryagin to determine an admissible control u^*. To this end, the following additional state variables are defined:

$$x_0 = \int_t^T \left(k + \frac{1}{2}u^2\right) d\tau, \qquad (19)$$
$$x_5 = \sin(x_2 - x_4) - (x_2 - x_4), \qquad x_6 = a_1, \qquad x_7 = a_2$$

The introduction of the extraneous state variables x_5, x_6, and x_7, whose motivation will become clear in the sequel, is a device frequently used in the application of the technique. With these additional state variables, the system equation is then given by

$$\dot{x}_0 = k + \tfrac{1}{2}u^2, \qquad \dot{x}_1 = V(x_2 - x_4 + x_5), \qquad \dot{x}_2 = x_3$$
$$\dot{x}_3 = -x_6x_3 - x_7x_3|x_3| - b_1x_4 + u, \qquad \dot{x}_4 = x_3 - b_2x_4 \qquad (20)$$
$$\dot{x}_5 = b_2x_4[\cos(x_2 - x_4) - 1], \qquad \dot{x}_6 = \dot{x}_7 = 0$$

It is desired to minimize $x_0(T)$ with the terminal condition $x_1(T) = x_2(T) = x_3(T) = x_4(T) = x_5(T) = 0$. The extraneous state variables x_6, x_7 are treated as free variables with $x_6(T)$ and $x_7(T)$ open. The Hamiltonian for this system is

$$h = p_0(k + \tfrac{1}{2}u^2) + Vp_1(x_2 - x_4 + x_5) + p_2x_3 + p_3(-x_6x_3 - x_7x_3|x_3| - b_1x_4 + u) + p_4(x_3 - b_2x_4) + b_2p_5x_4[\cos(x_2 - x_4) - 1] \qquad (21)$$

and the maximization of h with respect to u results in the control law u^*

$$u^* = -p_3/p_0 = p_3 \qquad (22)$$

where the adjoint variables p_i, $i = 0, 1, \ldots, 7$ satisfy

$$\dot{p}_0 = \dot{p}_1 = 0, \qquad \dot{p}_2 = -Vp_1 + b_2p_5x_4\sin(x_2 - x_4)$$
$$\dot{p}_3 = -p_2 + p_3x_6 + 2p_3x_7x_3 - p_4$$
$$\dot{p}_4 = Vp_1 + b_1p_3x_4 + b_2p_4 - b_2p_5[x_4\sin(x_2 - x_4) + \cos(x_2 - x_4) - 1] \qquad (23)$$
$$\dot{p}_5 = -Vp_1, \qquad \dot{p}_6 = p_3x_3, \qquad \dot{p}_7 = p_3x_3|x_3|$$

and the transversality conditions, which are $p_0(T) = -1$, $p_j(T) = 0$ for those values of j for which $x_j(T)$ are free.

Assuming that the original dynamic system (20) can be approximated by a simpler system with state $\mathbf{X}$ and control U and the adjoint state $\mathbf{P}$, where $\mathbf{X}$ is related to the original state x by an additive correction factor $\boldsymbol{\xi}$

$$\mathbf{x} = \mathbf{X} + \boldsymbol{\xi} \qquad (24)$$

then it was shown that if $\boldsymbol{\xi}$ is sufficiently small, the adjoint state $\mathbf{p}$ of the original system can also be expressed as the sum of the adjoint state $\mathbf{P}$ of the simplified system and a correction term $\boldsymbol{\psi}$, such that

$$\mathbf{p} = \mathbf{P} + \boldsymbol{\psi} = \mathbf{P} + M(\mathbf{X})\boldsymbol{\xi} \qquad (25)$$

where M is a symmetrical $(n + 1) \times (n + 1)$ matrix satisfying the matrix Riccati equation.

$$-(dM/dt) = MH_{XP} + H_{PX}M + MH_{PP}M + H_{XX} \qquad (26)$$

and the elements of coefficient matrices are given by

$$H_{XP} = (\partial^2h/\partial x_j\partial p_i)_{x=X,\ p=P}, \qquad H_{PX} = (\partial^2h/\partial p_j\partial x_i)_{x=X,\ p=P}$$
$$H_{PP} = (\partial^2h/\partial p_j\partial p_i)_{x=X,\ p=P}, \qquad H_{XX} = (\partial^2h/\partial x_j\partial x_i)_{x=X,\ p=P} \qquad (27)$$

Reference 1 describes several methods of obtaining analytical and approximate solutions of the Riccati equation.

For this technique to be useful the adjoint state $\mathbf{P}(\mathbf{X})$ of the simplified system must be determined as an explicit function of the state $\mathbf{X}$ of the simplified process. For this purpose, we define the simplified system by letting $x_4 = x_5 = x_6 = x_7 \equiv 0$ in (20), and thereupon obtain

$$\dot{X}_0 = k + \tfrac{1}{2}U^2, \qquad \dot{X}_1 = VX_2, \qquad \dot{X}_2 = \dot{X}_3, \qquad \dot{X}_3 = U,$$
$$\dot{X}_4 = \dot{X}_5 = \dot{X}_6 = \dot{X}_7 \equiv 0 \qquad (28)$$

The simplified dynamic system (28) represents a fictitious airplane with smooth surfaces flying at a constant angle of attack. An optimal feedback control U^* for the simplified system was obtained in Ref. 4 and is given by $U^* = P_3 = -(3/T^3)[(20/V)X_1 + 12X_2T + 3X_3T^2]$ or, in terms of the actual variables

$$C_{m\delta}\Delta\delta_s = -(3/T^3)[(20/V)\Delta y + 12T\Delta\theta + 3T^2\Delta\dot{\theta}] \qquad (29)$$

where the terminal time T is obtained from the least positive real solution of the following equation:

$$T^6 = (9/2k)[(20/V)X_1 + 8X_2T + X_3T^2]^2 = 0 \qquad (30)$$

This equation can be solved explicitly for $T = T(X_1, X_2, X_3)$. A closed form algorithm is derived in Ref. 4 and summarized in the appendix.

With the selection of the simplified system (28), the vectors $\mathbf{X}$, $\mathbf{P}$, $\boldsymbol{\xi}$, $\boldsymbol{\psi}$ are given by

$$\begin{aligned} \mathbf{X} &= [X_0, X_1, X_2, X_3, 0, 0, 0, 0] \\ \mathbf{P} &= [P_0, P_1, P_2, P_3, 0, 0, 0, 0] \\ \boldsymbol{\xi} &= [\xi_0, 0, 0, 0, x_4, x_5, x_6, x_7] \\ \boldsymbol{\psi} &= [\psi_0, \psi_1, \psi_2, \psi_3, p_4, p_5, p_6, p_7] \end{aligned} \qquad (31)$$

It is seen from (22) that the optimal control law u^* for the original system depends only on p_3. Thus, we conclude from

Table 1 Aircraft initial conditions

	Speed v, fps	Altitude $y(0)$, ft	Altitude rate $\dot{y}(0)$, fps	Angle of attack $\alpha(0)$, deg	Pitch $\theta(0)$, deg	Pitch rate $\dot{\theta}(0)$, deg/sec
Ideal	230	500	−17.0	4.7	0.4	0.02
Actual	230	400	−30.0	2.5	−5.0	0

(25) that the quasi-optimum control law u_q takes the form

$$
\begin{aligned}
u_q &= P_3 + \psi_3 \\
&= P_3 + m_{30}\xi_0 + m_{34}x_4 + m_{35}x_5 + m_{36}x_6 + m_{37}x_7 \qquad (32)
\end{aligned}
$$

where m_{ij} are elements of matrix M to be determined subsequently. The coefficient matrices H_{XP}, H_{PX}, H_{PP}, and H_{XX} for the Riccati equation (26) are now obtained from (21) and (27).

$$
H_{XP} = H'_{PX} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & V & 0 & -V & V & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -b_1 & 0 & -X_3 & -X_3|X_3| \\
0 & 0 & 0 & 1 & -b_2 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & b_2(\cos X_2 - 1) & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \qquad (33)
$$

$$
H_{PP} =
\begin{bmatrix}
P_3^2 & 0 & 0 & P_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
P_3 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix} \qquad (34)
$$

$$
H_{XX} =
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -P_3 & -2P_3|X_3| \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -P_3 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -2P_3|X_3| & 0 & 0 & 0 & 0
\end{bmatrix} \qquad (35)
$$

Because of the nonlinear terms, an explicit solution to the matrix Riccati equation (26) with time varying coefficient matrices given by (33), (34), and (35) is too complicated to obtain. We, therefore, approximate the exact solution by an asymptotic solution that can be solved by assuming $dM/dt = 0$. The resulting solutions to the pertinent correction factors in (32) are

$$
\begin{aligned}
m_{30} = 0, \quad m_{34} = -\tfrac{1}{2}m_{33}^2, \quad m_{35} = 0, \\
m_{36} = 2X_3, \quad m_{37} = 2X_3|X_3|
\end{aligned} \qquad (36)
$$

where m_{33} is obtained by solving the cubic equation

$$
m_{33}^3 - 4b_2m_{33}^2 + 4(b_1 + b_2^2)m_{33}^2 - 8b_1b_2 = 0 \qquad (37)
$$

Since b_1 and b_2 depend only on the aerodynamic coefficients, m_{33} is a real constant for any given aircraft. Furthermore, it can be shown that for physically meaningful values of b_1 and b_2, (37) yields only one real solution and is given by

$$
\begin{aligned}
m_{33} = \tfrac{2}{3}[b_2(\tfrac{9}{2}b_1 - b_2^2) + 3b_1(3b_1 - \tfrac{3}{4}b_2^2)^{1/2}]^{1/3} + \\
\tfrac{2}{3}[b_2(\tfrac{9}{2}b_1 - b_2^2) - 3b_1(3b_1 - \tfrac{3}{4}b_2^2)^{1/2}]^{1/3}
\end{aligned} \qquad (38)
$$

The quasi-optimum control law is now written as

$$
u_q = U^*_{X=x} - \tfrac{1}{2}m_{33}^2x_4 + 2x_3x_6 + 2x_3|x_3|x_7 \qquad (39)
$$

or, in terms of the actual variables,

$$
\begin{aligned}
C_{m\delta}\Delta\delta_q &= \\
&(1/C_{m\delta})\{-(3/T^3)[(20/V)\Delta y + 12T\Delta\theta + 3T^2\Delta\dot{\theta}] - \\
&\tfrac{1}{2}m_{33}^2\Delta\alpha - 2\Delta\dot{\theta}(C_{m\dot{\theta}} + C_{m\dot{\alpha}} + C_{m\dot{\theta}\dot{\theta}}|\Delta\dot{\theta}|)\} \qquad (40)
\end{aligned}
$$

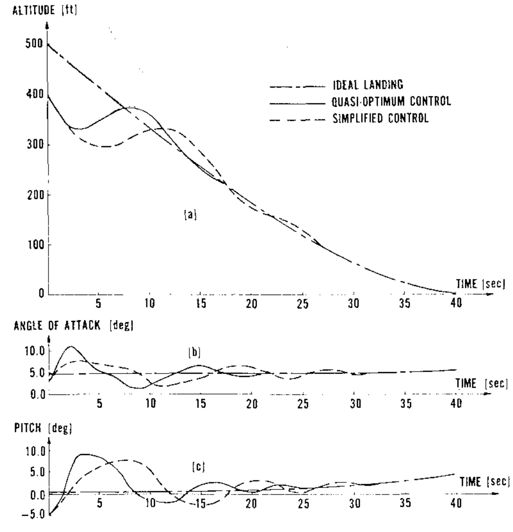


Fig. 3 Transient responses for $k = 0.005$.

4. System Performance

To evaluate the effectiveness of the quasi-optimum feedback control law as derived in the last section, the system transient responses were obtained by means of computer simulation. The aircraft parameters chosen for the numerical examples, roughly correspond to a present day jet airliner, and are as follows: $v = 230$ fps, $C_{m0} = 0.0$, $C_{m\dot{\theta}} = -0.5$, $C_{m\dot{\theta}\dot{\theta}} = -0.02$, $C_{m\alpha} = -0.93$, $C_{m\dot{\alpha}} = -0.207$, $C_{m\delta} = -0.694$, $C_{L0} = 0.083$, and $C_{L\alpha} = 0.696$. For the purpose of simulation study, an ideal descending route for the last 40.0 sec prior to touch-down, which approximately encompass part of the instrument-guided approach, the flare-out and the touch-down phases of a landing operation, is defined as

$$
\begin{aligned}
y^* &= -v\{0.08(\tau - 40.0) + 1.1175[1 - e^{-0.06264(\tau-40.0)}]\} \\
\varphi^* &= \sin^{-1}(v/y^*)
\end{aligned} \qquad (41)
$$

to provide a glide angle φ^* of $-4.5°$ during the instrument-guided approach and $-0.6°$ at the touch-down.

The resulting simplified and quasi-optimum control laws, starting from 40.0 sec prior to touch-down, are obtained from (6), (29), and (40) as

$$
\begin{aligned}
C_{m\delta}\delta_s(\tau) &= C_{m\delta}[\delta^*(\tau) + \Delta\delta_s] \\
&= \{1.437\dddot{\varphi}^* + 2.016\ddot{\varphi}^* + 1.836\dot{\varphi}^* + \\
&\quad 0.02(\dot{\varphi}^* + 1.437\ddot{\varphi}^*)|\dot{\varphi}^* + 1.437\ddot{\varphi}^*| + 0.0744\} - \\
&\quad (3/T^3)[(20/V)\Delta y + 12T\Delta\theta + 3T^2\Delta\dot{\theta}] \qquad (42)
\end{aligned}
$$

and

$$
\begin{aligned}
C_{m\delta}\delta_q(\tau) &= C_{m\delta}[\delta^*(\tau) + \Delta\delta_q] = \\
&C_{m\delta}\delta_s(\tau) - \{0.107\Delta\alpha - (1.414 + 0.04|\Delta\dot{\theta}|)\Delta\dot{\theta}\} \qquad (43)
\end{aligned}
$$

Note that the only difference between δ_s and δ_q is that the latter provides additional damping for angle of attack and pitch rate errors.

The initial conditions as shown in Table 1 are chosen to depict a worst-case situation for the aircraft to enter into the landing operation as defined by the ideal trajectory.

Figures 2 and 3 show the transient responses of the aircraft for $k = 0.001$ and $k = 0.005$, respectively. A comparison of these figures shows that the system response tends to be faster and more oscillatory for the higher value of k, and slower and smoother for the lower values of k.

For $k = 0.001$, it is seen from Fig. 2 that the quasi-optimum (simplified) control system provides a maximum angle-of-attack deviation ($\Delta\alpha$max) of 4°(2°) and a maximum pitch angle deviation ($\Delta\theta$max) of 6°(6°) that are considered within

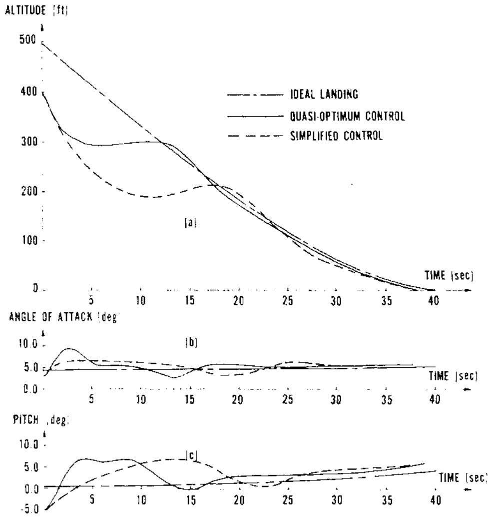


Fig. 4 Transient responses for $k = 0.001$ with steady downwind disturbance.

allowable ranges. A further decrease in k also decreases the values of $\Delta\alpha$max and $\Delta\theta$max with more sluggish responses.

On the other hand, an increase in k, as it is seen from Fig. 3 with $k = 0.005$, the values of $\Delta\alpha$max and $\Delta\theta$max also increase to 7°(3°) and 8°(6°), respectively. The responses are faster and the overshoots are higher. An appropriate value of k, which gives a compromised response, therefore, should be around 0.001.

The transient time T required for the altitude error $\Delta y(t)$ to reach within ±2.0 ft, the values of performance index S evaluated up to T and the ranges of elevator deflection δ are tabulated in Table 2.

Thus, the quasi-optimum control does provide an acceptable performance satisfying all pertinent physical conditions. The behavior of the control system in response to steady downwind disturbances was also investigated. For this study, it is assumed that the wind has a constant velocity component perpendicular to the flight trajectory and that its velocity component tangential to the trajectory is offset by an engine throttle control so that the aircraft forward speed remains constant. Under these assumptions, the wind disturbance is conveniently represented by a constant angle η added to the angle of attack α, i.e., the tangent of η is equal to the ratio of the wind component perpendicular to the flight path and the aircraft forward speed. Let α_w be the aircraft

Table 2 Comprison of performance

	$k = 0.001$ Quasi-optimum	$k = 0.001$ Simplified	$k = 0.005$ Quasi-optimum	$k = 0.005$ Simplified
Transient time T, sec	20.0	31.0	18.0	26.0
Performance measure S	0.037	0.043	0.128	0.155
Range of elevator deflection δ, deg	−15.0 to +3.0	−11.0 to +1.0	−20.0 to +3.0	−16.0 to +1.0

angle of attack when the wind disturbance is present, then we must have $\alpha = \alpha_w - \eta$ and $\Delta\alpha = \Delta\alpha_w - \eta$.

Substituting above relation to (12) and subsequently to (14–17) and then applying the control law of (40), it is observed that the transient responses remain similar to those obtained without wind disturbance with the exception of some steady-state altitude deviation. For a downward wind with a velocity about 10% of the aircraft forward speed, it was found that the steady-state altitude deviation ranges from −3 ft to −10 ft for various values of k. For $k = 0.001$, Fig. 4 shows that the steady state altitude error for the quasi-optimum control system is approximately −4 ft and, as a result, the aircraft touches the ground at $\tau = 38.5$ sec with glide angle −0.7°, and descending rate −3.5 fps., which are well within safety ranges. Pulsed downwind disturbances were also considered in the simulation with a result of less effect on the trajectories than that of steady disturbance. Since only the longitudinal motions were considered in this investigation, the effects due to other disturbances, such as crosswind and rolling gust, can not be realistically simulated. Simulation studies for various forward speed v were also performed with similar results.

5. Conclusion

It is demonstrated that the multiple performance requirements for the design of an elevator controller can be met by a proper choice of a single factor k, defined to achieve a compromise between a fast but rough response and a slow but smooth response. The resulting trajectories shown in this paper resemble closely the trajectories of a similar case obtained by Ellert and Merriam[3] using an entirely different design approach. In view of the highly restrictive design specifications, the similarity of the results are not surprising,

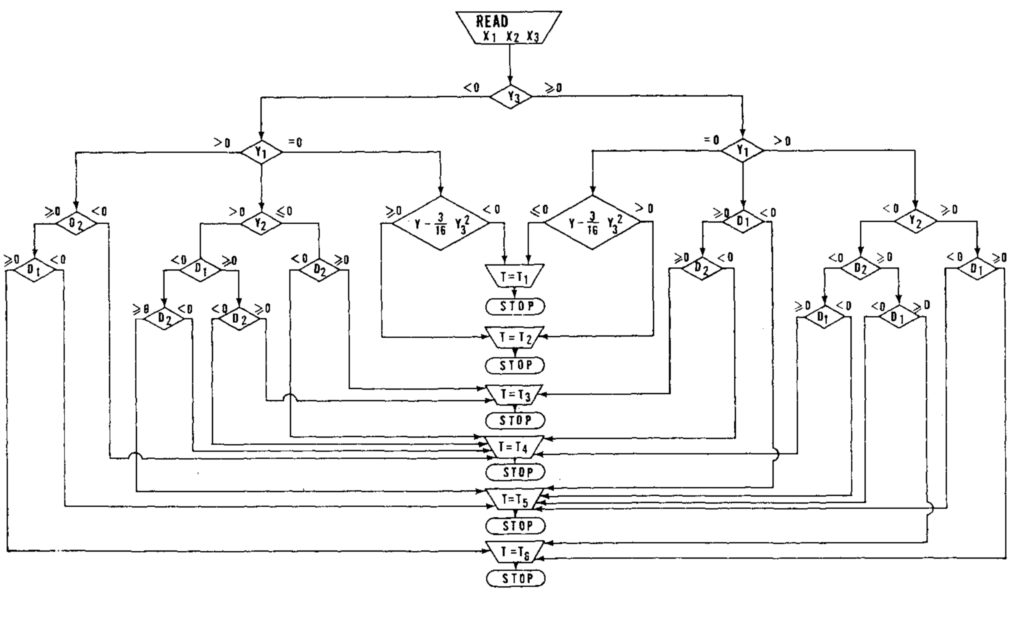


Fig. 5 Flowchart for computing terminal time for simplified system.

since the set of admissible trajectories for a given initial condition is rather limited irrespective of the control techniques employed.

One significant advantage of using the present formulation is the simplicity it provides in designing the control system. With the choice of performance index (11) and the fixed terminal condition of zero deviations, application of Friedland's quasi-optimum control technique results in a relatively simple control law. In view of the satisfactory performance achieved in this study, it would appear that further work is warranted on extending the approach to the complete degree-of-freedom aircraft landing control problem.

Appendix

The purpose of this appendix is to summarize the methods for solution (30) for T, which is positive real and provides a minimum value to the performance index

$$S_s = \int_t^T \left(k + \frac{1}{2} U^2\right) d\tau$$

A detailed account is rather tedious and is given in Ref. 4. For convenience, the algorithm for calculating the nonunique expressions $T = T(X_1, X_2, X_3)$ is shown by a flowchart in Fig. 5, where

$$Y_1 = [5/V(2k)^{1/2}]X_1,\ Y_2 = [2/(2k)^{1/2}]X_2,\ Y_3 = [1/(2k)^{1/2}]X_3$$

$$D_1(Y_1, Y_2, Y_3) = -64Y_2^3 - 12Y_3^2Y_2^2 + 72Y_1Y_2Y_3 + 36Y_1^2 + 12Y_1Y_3^3$$

$$D_2(Y_1, Y_2, Y_3) = 64Y_2^3 + 12Y_3^2Y_2^2 - 72Y_1Y_2Y_3 - 36Y_1^2 - 12Y_1Y_{33}$$

$$T_1 = \tfrac{1}{2}[-3Y_3 + (9Y_3^2 - 48Y_2)^{1/2}]$$

$$T_2 = \tfrac{1}{2}[3Y_3 + (9Y_3^2 + 48Y_2)^{1/2}]$$

$$T_3 = (-Y_3^3 + 6Y_2Y_3 - 6Y_1 + D_2^{1/2})^{1/3} + (-Y_3^3 + 6Y_2Y_3 - 6Y_1 - D_2^{1/2})^{1/3} - Y^3$$

$$T_4 = 2(Y_3^2 - 4Y_2)^{1/2}\cos(\phi_2/3) - Y_3; \quad \phi_2 = \cos^{-1}[(Y_3^2 + 6Y_2Y_3 - 6Y_1)/(Y_3^2 - 4Y_2)^{3/2}]$$

$$T_5 = 2(Y_3^2 + 4Y_2)^{1/2}\cos(\phi_1/3) + Y_3; \quad \phi_1 = \cos^{-1}[(Y_3^2 + 6Y_2Y_3 + 6Y_1)/(Y_3^2 + 4Y_2)^{3/2}]$$

$$T_6 = (Y_3^3 + 6Y_2Y_3 + 6Y_1 + D_1^{1/2})^{1/3} + (Y_3^3 + 6Y_2Y_3 + 6Y_1 - D_1^{1/2})^{1/3} + Y_3$$

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